Lecturered By Dr. H.K. Yadav.
Deptt. of Mathematics
Marwari College, Darbhanga.
B.sc-part-II(H), paper-III, Date- 29-04-2020
Differential Calculus

Q.1. Prove that $\frac{x}{a}+\frac{y}{b}=1$ touches the curve $y=be^{-\frac{x}{a}}$ at the point where the curve crosses the axis of y

Soln: We know that the equation of the y-axis is $x=0$.

The point at which the curve crosses the axis of y is $(0,b)$

Now, the eqn of the tangent at $(0,b)$ to the curve is

$$Y-y=(X-x)\frac{dy}{dx}, \text{ where } y=b,\ x=0$$

But $\frac{dy}{dx}$ at $y=b$, $x=0$ is $\left[-\frac{b}{a}e^{-\frac{x}{a}}\right]_{\substack{y=b\\x=0}}=-\frac{b}{a}$

$\therefore$ The eqn of the tangent becomes

$$Y-b=(X-0)\left(-\frac{b}{a}\right) \Rightarrow Y-b=-\frac{b}{a}X$$

$$\Rightarrow \frac{X}{a}+\frac{Y}{b}=1$$

If we take $(x,y)$ instead of $(X,Y)$ for the current co-ordinates on the tangent, the eqn becomes $\frac{x}{a}+\frac{y}{b}=1$ Proved.

Q.2. Find the abscissa of the point on the curve $ay^2=x^3$, the normal at which cuts off equal intercepts from the co-ordinates axes.

Soln: Here, it is given that $f(x,y)\equiv ay^2-x^3=0$

$\therefore$ Eqn of the normal at $(x,y)$ to the curve is

$$\frac{X-x}{-3x^2}=\frac{Y-y}{2ay}$$

Rest part of Q.2.

$$\frac{X}{3x^2} + \frac{Y}{2ay} = \left(\frac{x}{3x^2} + \frac{y}{2ay}\right)$$

$$\Rightarrow \frac{X}{3x^2\left(\frac{1}{3x}+\frac{1}{2a}\right)} + \frac{Y}{2ay\left(\frac{1}{3x}+\frac{1}{2a}\right)} = 1$$

Not if it cuts off equal intercepts from the co-ordinate axes.

$$3x^2\left(\frac{1}{3x}+\frac{1}{2a}\right) = 2ay\left(\frac{1}{3x}+\frac{1}{2a}\right)$$

$\therefore$ either $3x^2 = 2ay \Rightarrow \frac{1}{3x}+\frac{1}{2a}=0$

Now, if $\frac{1}{3x}+\frac{1}{2a}=0$, then $x=-\frac{2a}{3}$. But then the normal passes through the origin, for the eqn. becomes $\frac{X}{3x^2}+\frac{Y}{2ay}=0$. Then, the case is trivial, the eqn intercepts being zero.

Excluding this case we have $3x^2 = 2ay$.

Put $ay^2 = x^3$.

Solving the two eqns

$$x^3 = a\left(\frac{3x^2}{2a}\right)^2 \Rightarrow 4a^2x^3 = 9ax^4$$

$\therefore$ $x^3(4a-9x)=0$; either $x=0$ or $x=\frac{4a}{9}$.

But when $x=0$, $y=0$. Then the normal passes through $(0,0)$ and ~~cut~~ cutting zero intercepts from each axis.

Hence, the required abscissa $=\frac{4a}{9}$.